Pre-Calculus/Trigonometry

In this technological age, mathematics is more important than ever. When students leave school, they are more and more likely to use mathematics in their work and everyday lives — operating computer equipment, planning timelines and schedules, reading and interpreting data, comparing prices, managing personal finances, and completing other problem-solving tasks. What they learn in mathematics and how they learn it will provide an excellent preparation for a challenging and ever-changing future.

The state of Indiana has established the following mathematics standards to make clear to teachers, students, and parents what knowledge, understanding, and skills students should acquire in Pre-Calculus/Trigonometry:

Standard 1 — Relations and Functions

Students recognize and graph polynomial, rational, algebraic, and absolute value functions and use them to solve word problems. They understand the concepts of domain, range, intercept, zero, pole, asymptote, and point of discontinuity. They define and find inverse functions, describe symmetries of graphs, and apply transformations to functions. They understand the concept of defining a function parametrically and apply it to drawing graphs. They write equations of conic sections in standard form to find their geometric properties.

Standard 2 — Logarithmic and Exponential Functions

Students solve word problems involving logarithmic and exponential functions. They draw and analyze graphs of logarithmic and exponential functions, including finding domain, range, intercepts, and asymptotes. They define and find inverse functions for both logarithmic and exponential functions.

Standard 3 — Trigonometry in Triangles

Students understand how trigonometric functions relate to right triangles and solve word problems involving right and oblique triangles. They understand and apply the laws of sines and cosines. They use trigonometry to find the area of a triangle from two sides and the included angle.

Standard 4 — Trigonometric Functions

Students extend the definitions of the trigonometric functions beyond right triangles using the unit circle and they measure angles in radians as well as degrees. They draw and analyze graphs of trigonometric functions (including finding period, amplitude, and phase shift) and use them to solve word problems. They define and graph inverse trigonometric functions and find values of both trigonometric and inverse trigonometric functions. They also relate the slope of a line to the tangent of the angle the line makes with the x-axis.

Standard 5 — Trigonometric Identities and Equations

Students know basic trigonometric identities derived from the definitions and use them to prove other results. In particular, they understand and use the addition, double-angle, and half-angle formulas. They solve trigonometric equations and apply the equations to word problems.

Standard 6 — Polar Coordinates and Complex Numbers

Students define and use polar coordinates, understanding their relationship with Cartesian coordinates. They translate equations in Cartesian coordinates into polar coordinates and graph equations in the polar coordinate plane. They understand complex numbers and convert them to trigonometric form. They multiply complex numbers in trigonometric form and prove and use De Moivre's Theorem.

Standard 7 — Sequences and Series

Students prove the formulas for the sums of arithmetic series and for finite and infinite geometric series, using summation notation and applying the results to word problems. They understand the concept of recursion and define sequences using it. They develop the concept of the limit of a sequence or a function and apply it to problems of convergence and divergence.

Standard 8 — Data Analysis

Students understand the median fit and least squares regression methods and apply them to linear modeling. They calculate and interpret correlation coefficients, using them to evaluate lines of best fit. They model data with various nonlinear functions, such as quadratic, exponential, and power functions.

Standard 9 — Mathematical Reasoning and Problem Solving

In a general sense, mathematics is problem solving. In all of their mathematics, students use problem-solving skills: they choose how to approach a problem, they explain their reasoning, and they check their results. At this level, students apply these skills to justifying the steps in simplifying functions and solving equations and to deciding whether algebraic statements are true. They also learn how to use the mathematical induction to prove results.

As part of their instruction and assessment, students should also develop the following learning skills by Grade 12 that are woven throughout the mathematics standards:

Communication

The ability to read, write, listen, ask questions, think, and communicate about math will develop and deepen students' understanding of mathematical concepts. Students should read text, data, tables, and graphs with comprehension and understanding. Their writing should be detailed and coherent, and they should use correct mathematical vocabulary. Students should write to explain answers, justify mathematical reasoning, and describe problem-solving strategies.

Representation

The language of mathematics is expressed in words, symbols, formulas, equations, graphs, and data displays. The concept of one-fourth may be described as a quarter, ¼, one divided by four, 0.25, ⅛ + ⅛, 25 percent, or an appropriately shaded portion of a pie graph. Higher-level mathematics involves the use of more powerful representations: exponents, logarithms, π, unknowns, statistical representation, algebraic and geometric expressions. Mathematical operations are expressed as representations: +, =, divide, square. Representations are dynamic tools for solving problems and communicating and expressing mathematical ideas and concepts.

Connections

Connecting mathematical concepts includes linking new ideas to related ideas learned previously, helping students to see mathematics as a unified body of knowledge whose concepts build upon each other. Major emphasis should be given to ideas and concepts across mathematical content areas that help students see that mathematics is a web of closely connected ideas (algebra, geometry, the entire number system). Mathematics is also the common language of many other disciplines (science, technology, finance, social science, geography) and students should learn mathematical concepts used in those disciplines. Finally, students should connect their mathematical learning to appropriate real-world contexts.

Pre-Calculus/Trigonometry

Pre-Calculus and Trigonometry are sometimes taught separately as one-semester courses, but are more often taught as a single course. If taught separately, the courses should consist of the following standards:

Pre-Calculus

- Standard 1 Relations and Functions
- Standard 2 Logarithmic and Exponential Functions
- Standard 7 Sequences and Series
- Standard 8 Data Analysis
- Standard 9 Mathematical Reasoning and Problem Solving

Trigonometry

- Standard 3 Trigonometry in Triangles
- Standard 4 Trigonometric Functions
- Standard 5 Trigonometric Identities and Equations
- Standard 6 Polar Coordinates and Complex Numbers
- Standard 9 Mathematical Reasoning and Problem Solving

PC

Standard 1

Relations and Functions

Students use polynomial, rational, and algebraic functions to write functions and draw graphs to solve word problems, to find composite and inverse functions, and to analyze functions and graphs. They analyze and graph circles, ellipses, parabolas, and hyperbolas.

PC.1.1 Recognize and graph various types of functions, including polynomial, rational, algebraic, and absolute value functions. Use paper and pencil methods and graphing calculators.

Example: Draw the graphs of the functions $y = x^5 - 2x^3 - 5x^2$, $y = \frac{2x-1}{3x+2}$, and $y = \sqrt{(x+2)(x-5)}$.

PC.1.2 Find domain, range, intercepts, zeros, asymptotes, and points of discontinuity of functions. Use paper and pencil methods and graphing calculators.

Example: Let $R(x) = \frac{1}{\sqrt{x-2}}$. Find the domain of $R(x)$ — i.e., the values of x for which $R(x)$ is defined. Also find the range, zeros, and asymptotes of $R(x)$.

PC.1.3 Model and solve word problems using functions and equations.

Example: You are on the committee for planning the prom and need to decide what to charge for tickets. Last year you charged $5.00 and 400 people bought tickets. Earlier experiences suggest that for every 10¢ decrease in price you will sell 50 extra tickets. Use a spreadsheet and write a function to show how the amount of money in ticket sales depends on the number of 10¢ decreases in price. Construct a graph that shows the price and gross receipts. What is the optimum price you should set for the tickets?

PC.1.4 Define, find, and check inverse functions.

Example: Find the inverse function of $h(x) = (x-2)^3$.

PC.1.5 Describe the symmetry of the graph of a function.

Example: Describe the symmetries of the functions x, x^2, x^3, and x^4.

PC.1.6 Decide if functions are even or odd.

Example: Is the function $\tan x$ even, odd, or neither? Explain your answer.

PC.1.7 Apply transformations to functions.

Example: Explain how you can obtain the graph of $g(x) = -|2(x+3)^2 - 2|$ from the graph of $f(x) = x^2$.

PC.1.8 Understand curves defined parametrically and draw their graphs.

Example: Draw the graph of the function $y = f(x)$, where $x = 3t + 1$ and $y = 2t^2 - 5$ for a parameter t.

PC.1.9 Compare relative magnitudes of functions and their rates of change.

Example: Contrast the growth of $y = x^2$ and $y = 2^x$.

PC.1.10 Write the equations of conic sections in standard form (completing the square and using translations as necessary), in order to find the type of conic section and to find its geometric properties (foci, asymptotes, eccentricity, etc.).

Example: Write the equation $x^2 + y^2 - 10x - 6y - 25 = 0$ in standard form. Decide what kind of conic it is and find its foci, asymptotes, and eccentricity as appropriate.

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Standard 2

Logarithmic and Exponential Functions

Students solve word problems involving logarithmic and exponential functions. They draw and analyze graphs and find inverse functions.

PC.2.1 Solve word problems involving applications of logarithmic and exponential functions.

Example: The amount A gm of a radioactive element after t years is given by the formula $A(t) = 100\,e^{-0.02t}$. Find t when the amount is 50 gm, 25 gm, and 12.5 gm. What do you notice about these time periods?

PC.2.2 Find the domain, range, intercepts, and asymptotes of logarithmic and exponential functions.

Example: For the function $L(x) = \log_{10}(x - 4)$, find its domain, range, x-intercept, and asymptote.

PC.2.3 Draw and analyze graphs of logarithmic and exponential functions.

Example: In the last example, draw the graph of $L(x)$.

PC.2.4 Define, find, and check inverse functions of logarithmic and exponential functions.

Example: Find the inverse of $f(x) = 3e^{2x}$.

Standard 3

Trigonometry in Triangles

Students define trigonometric functions using right triangles. They solve word problems and apply the laws of sines and cosines.

PC.3.1 Solve word problems involving right and oblique triangles.

Example: You want to find the width of a river that you cannot cross. You decide to use a tall tree on the other bank as a landmark. From a position directly opposite the tree, you measure 50 m along the bank. From that point, the tree is in a direction at 37° to your 50 m line. How wide is the river?

PC.3.2 Apply the laws of sines and cosines to solving problems.

Example: You want to fix the location of a mountain by taking measurements from two positions 3 miles apart. From the first position, the angle between the mountain and the second position is 78°. From the second position, the angle between the mountain and the first position is 53°. How far is the mountain from each position?

PC.3.3 Find the area of a triangle given two sides and the angle between them.

Example: Calculate the area of a triangle with sides of length 8 cm and 6 cm enclosing an angle of 60°.

Standard 4

Trigonometric Functions

Students define trigonometric functions using the unit circle and use degrees and radians. They draw and analyze graphs, find inverse functions, and solve word problems.

PC.4.1 Define sine and cosine using the unit circle.

Example: Find the acute angle A for which $\sin 150^\circ = \sin A$.

PC.4.2 Convert between degree and radian measures.

Example: Convert 90°, 45°, and 30° to radians.

PC.4.3 Learn exact sine, cosine, and tangent values for 0, $\pi/2$, $\pi/3$, $\pi/4$, $\pi/6$, and multiples of π. Use those values to find other trigonometric values.

Example: Find the values of $\cos \pi/2$, $\tan 3\pi/4$, $\csc 2\pi/3$, $\sin^{-1} \sqrt{3}/2$, and $\sin 3\pi$.

PC.4.4 Solve word problems involving applications of trigonometric functions.

Example: In Indiana, the day length in hours varies through the year in a sine wave. The longest day of 14 hours is on Day 175 and the shortest day of 10 hours is on Day 355. Sketch a graph of this function and find its formula. Which other day has the same length as July 4?

PC.4.5 Define and graph trigonometric functions (i.e., sine, cosine, tangent, cotangent, secant, cosecant).

Example: Graph $y = \sin x$ and $y = \cos x$, and compare their graphs.

PC.4.6 Find domain, range, intercepts, periods, amplitudes, and asymptotes of trigonometric functions.

Example: Find the asymptotes of $\tan x$ and find its domain.

PC.4.7 Draw and analyze graphs of translations of trigonometric functions, including period, amplitude, and phase shift.

Example: Draw the graph of $y = 5 + \sin (x - \pi/3)$.

PC.4.8 Define and graph inverse trigonometric functions.

Example: Graph $f(x) = \sin^{-1} x$.

PC.4.9 Find values of trigonometric and inverse trigonometric functions.

Example: Find the values of $\sin \pi/2$ and $\tan^{-1} \sqrt{3}$.

PC.4.10 Know that the tangent of the angle that a line makes with the x-axis is equal to the slope of the line.

Example: Use a right triangle to show that the slope of a line at 135° to the x-axis is -1.

PC.4.11 Make connections between right triangle ratios, trigonometric functions, and circular functions.

Example: Angle A is a 60° angle of a right triangle with a hypotenuse of length 14 and a shortest side of length 7. Find the exact sine, cosine, and tangent of angle A. Find the real numbers x, $0 < x < 2\pi$, with exactly the same sine, cosine, and tangent values.

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Standard 5

Trigonometric Identities and Equations

Students prove trigonometric identities, solve trigonometric equations, and solve word problems.

PC.5.1 Know the basic trigonometric identity $\cos^2 x + \sin^2 x = 1$ and prove that it is equivalent to the Pythagorean Theorem.

Example: Use a right triangle to show that $\cos^2 x + \sin^2 x = 1$.

PC.5.2 Use basic trigonometric identities to verify other identities and simplify expressions.

Example: Show that $\frac{\tan^2 x}{1 + \tan^2 x} = \sin^2 x$.

PC.5.3 Understand and use the addition formulas for sines, cosines, and tangents.

Example: Prove that $\sin(A + B) = \sin A \cos B + \cos A \sin B$ and use it to find a formula for $\sin 2x$.

PC.5.4 Understand and use the half-angle and double-angle formulas for sines, cosines, and tangents.

Example: Prove that $\cos^2 x = \frac{1}{2} + \frac{1}{2}\cos 2x$.

PC.5.5 Solve trigonometric equations.

Example: Solve $3 \sin 2x = 1$ for x between 0 and 2π.

PC.5.6 Solve word problems involving applications of trigonometric equations.

Example: In the example about day length in Standard 4, for how long in winter is there less than 11 hours of daylight?

Standard 6

Polar Coordinates and Complex Numbers

Students define polar coordinates and complex numbers and understand their connection with trigonometric functions.

PC.6.1 Define polar coordinates and relate polar coordinates to Cartesian coordinates.

Example: Convert the polar coordinates $(2, \frac{\pi}{3})$ to (x, y) form.

PC.6.2 Represent equations given in rectangular coordinates in terms of polar coordinates.

Example: Represent the equation $x^2 + y^2 = 4$ in terms of polar coordinates.

PC.6.3 Graph equations in the polar coordinate plane.

Example: Graph $y = 1 - \cos\theta$.

PC.6.4 Define complex numbers, convert complex numbers to trigonometric form, and multiply complex numbers in trigonometric form.

Example: Write $3 + 3i$ and $2 - 4i$ in trigonometric form and then multiply the results.

PC.6.5 State, prove, and use De Moivre's Theorem.

Example: Simplify $(1 - i)^{23}$.

Standard 7

Sequences and Series

Students define and use arithmetic and geometric sequences and series, understand the concept of a limit, and solve word problems.

PC.7.1 Understand and use summation notation.

Example: Write the terms of $\sum_{1}^{5} n^2$.

PC.7.2 Find sums of infinite geometric series.

Example: Find the sum of $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \ldots$.

PC.7.3 Prove and use the sum formulas for arithmetic series and for finite and infinite geometric series.

Example: Prove that $a + ar + ar^2 + ar^3 + ar^4 + \ldots = a / (1 - r)$.

PC.7.4 Use recursion to describe a sequence.

Example: Write the first five terms of the Fibonacci sequence with $a_1 = 1$, $a_2 = 1$, and $a_n = a_{n-1} + a_{n-2}$ for $n \geq 3$.

PC.7.5 Understand and use the concept of limit of a sequence or function as the independent variable approaches infinity or a number. Decide whether simple sequences converge or diverge.

Example: Find the limit as $n \to \infty$ of the sequence $\frac{2n - 1}{3n + 2}$ and the limit as $x \to 5$ of the function $\frac{x^2 - 5^2}{x - 5}$.

PC.7.6 Solve word problems involving applications of sequences and series.

Example: You put $100 in your bank account today, and then each day put half the amount of the previous day (always rounding to the nearest cent). Will you ever have $250 in your account?

Standard 8

Data Analysis

Students model data with linear and nonlinear functions.

PC.8.1 Find linear models using the median fit and least squares regression methods. Decide which model gives a better fit.

Example: Measure the wrist and neck size of each person in your class and make a scatterplot. Find the median fit line and the least squares regression line. Which line is a better fit? Explain your reasoning.

PC.8.2 Calculate and interpret the correlation coefficient. Use the correlation coefficient and residuals to evaluate a "best-fit" line.

Example: Calculate and interpret the correlation coefficient for the linear regression model in the last example. Graph the residuals and evaluate the fit of the linear equation.

PC.8.3 Find a quadratic, exponential, logarithmic, power, or sinusoidal function to model a data set and explain the parameters of the model.

Example: Drop a ball and record the height of each bounce. Make a graph of the height (vertical axis) versus the bounce number (horizontal axis). Find an exponential function of the form $y = a \bullet b^x$ that fits the data and explain the implications of the parameters a and b in this experiment.

Standard 9

Mathematical Reasoning and Problem Solving

Students use a variety of strategies to solve problems.

PC.9.1 Use a variety of problem-solving strategies, such as drawing a diagram, guess-and-check, solving a simpler problem, examining simpler problems, and working backwards.

Example: The half-life of carbon-14 is 5,730 years. The original concentration of carbon-14 in a living organism was 500 grams. How might you find the age of a fossil of that living organism with a carbon-14 concentration of 140 grams?

PC.9.2 Decide whether a solution is reasonable in the context of the original situation.

Example: John says the answer to the problem in the first example is about 10,000 years. Is his answer reasonable? Why or why not?

Students develop and evaluate mathematical arguments and proofs.

PC.9.3 Decide if a given algebraic statement is true always, sometimes, or never (statements involving rational or radical expressions, trigonometric, logarithmic or exponential functions).

Example: Is the statement $\sin 2x = 2 \sin x \cos x$ true for all x, for some x, or for no x? Explain your answer.

PC.9.4 Use the properties of number systems and order of operations to justify the steps of simplifying functions and solving equations.

Example: Simplify $\left(\frac{5}{x-2}+\frac{2}{x+3}\right) \div \left(\frac{1}{x+3}+\frac{7}{x-2}\right)$, explaining why you can take each step.

PC.9.5 Understand that the logic of equation solving begins with the assumption that the variable is a number that satisfies the equation, and that the steps taken when solving equations create new equations that have, in most cases, the same solution set as the original. Understand that similar logic applies to solving systems of equations simultaneously.

Example: A student solving the equation $x + \sqrt{x} - 30 = 0$ comes up with the solution set $\{25, 36\}$. Explain why $\{25, 36\}$ is not the solution set to this equation, and why the "check" step is essential in solving the equation.

PC.9.6 Define and use the mathematical induction method of proof.

Example: Prove De Moivre's Theorem using mathematical induction.

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